Robust Topology Optimization of an IPM Machine using Sensitivity Analysis

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Abstract — This paper presents a robust topology optimization formulation based on topological sensitivity analysis. This method is applied to the design of an IPM (interior permanent magnet) motor. The topological gradient is computed for both the source design problem and the material design problem. The shape and the topology are being optimized simultaneously thus neither a solidification process nor a topology smoother is required. The uncertainties in the design variables are taken into account using a robust formulation of the objective function.

I. INTRODUCTION

IPM (interior permanent magnet) motors have been used in many industries, such as automotive and home appliances. The advantages of the IPM include high efficiency and easy speed control. In order to improve the performance and reduce the running noise of the machine, we must reduce the cogging torque of the motor. Therefore, this becomes a critical problem in the design of an IPM motor.

In recent years, the FEM (finite element method) has been applied in the analysis and design of IPM motors. In [1], the authors have built a FEM model considering the rotation of the motor, and conducted the design of the motor using experimental design method. The shape of the surface of the iron core in an IPM motor has been optimized to reduce the cogging torque using CDSA (Continuum Design Sensitivity Analysis) in reference [2]. While in [2], only the surface of the rotor is allowed to change within a small range, Takahashi *et al.* proposed a new design scheme using the ON/OFF method in order to find new magnetic circuits [3]. Two designs have been presented in [3], one is the surface optimization and the other is the design of the field barrier using topology optimization. However, the resultant surfaces of the barrier from the conventional topology optimization are not smooth; therefore, the design is not practical for manufacture. In addition, the design of an IPM machine is not a single target problem. We must increase the torque to meet the minimal torque requirement while the other parameters are being optimized to reduce the cogging torque.

In this paper, a robust topology optimization of the IPM is proposed using topological sensitivity analysis. Both the OMD (optimal material distribution) and OSD (optimal source distribution) problems are considered. A conceptual design is started from scratch in order to increase the output torque of the machine. Also the method is applied to reduce the torque ripple of a classical layout of the IPM motor.

II. METHODOLOGY

The topological shape design [4] [5] method is applied to solve the topology optimization problem of an IPM machine design. During the process, the shape and topology optimization are carried out simultaneously. A robust objective function is used for managing uncertainties.

A. Shape Design Sensitivity Analysis

The shape design sensitivity formula is expressed as a line integral of a function of the field values computed from primary and adjoint field solutions [6],

$$\frac{dF}{dp} = \int_{\gamma} G(A,\lambda) n \cdot d\Gamma. \tag{1}$$

For magnetostatic problems,

$$G(A,\lambda) = \nabla \times \lambda_2 \cdot [(\nu_1 - \nu_2)\nabla \times A_1 + (M_2 - M_1)] + (J_2 - J_1)\lambda_2 \tag{2}$$

where all values with subscript indices 1 and 2 correspond to the two sub-domains Ω_1 and Ω_2 containing different materials, γ is the boundary between Ω_1 and Ω_2, ν is the material reluctivity, A and λ are the state and adjoint variables, M is the permanent magnetization, J is the current density, and n is an unit vector normal to the boundary γ.

B. Topological Gradient

The topological gradient (TG) provides the information on the opportunity to create a small hole (inhomogeneity) with a radius d, centered at r, in the domain Ω:

$$TG(r) = \lim_{d\to 0} \frac{\Psi_{obj}(\Omega \backslash B(r,d) - \Psi_{obj}}{\delta(\Omega)}, \tag{3}$$

where Ψ is an arbitrary objective function.

TG(r) is solved using an asymptotic expansion [4] [5] and it can be linked with the shape sensitivity, in 2-D, as:

$$TG(r) = 2G(A,\lambda). \tag{4}$$

In order to minimize the objective function Ψ, the TG must be greater than zero. Therefore topology changes take place in the areas with the highest TG values.

C. Material updating Scheme

The topology optimization of an IPM involves two kinds of design problems, the OMD (optimal material distribution) and the OSD (optimal source distribution). In the OSD, we need to decide where to place the source, i.e. the PM (permanent magnet), in the iron core. The material swapping happens between iron or air and PM, thus the topological gradient is given as:

$$TG = 2\nabla \times \lambda_2 \cdot [(\nu_1 - \nu_2)\nabla \times A_1 + (M_2 - M_1)]. \tag{5}$$

In OMD, one must choose a material between iron and air, thus the TG is calculated as:

$$TG = 2(\nu_1 - \nu_2)\nabla \times A_1 \cdot \nabla \times \lambda_2. \tag{6}$$

D. *Robust Topology Optimization*

In topological shape optimization, the topology and shape are being optimized at the same time, and the uncertainties in the design variables need to be taken into account in both optimization cycles. A robust objective function is formulated based on the worst performance of the function value due to the perturbation of the design variable x and is given as:

$$f_w = \max_{\xi \in U(x)} f(\xi). \qquad (7)$$

where U is the uncertainty set expressed as:

$$U(x_0) = \{\xi \in R^n : x_0 - \Delta \leq \xi \leq x_0 + \Delta\}. \qquad (8)$$

where the vector $\Delta = \{\Delta_1\ \Delta_2\ \ldots\ \Delta_n\}$ represents the largest variation to the nominal value x_0 of the design variables.

Hence we can obtain a robust topological gradient [7] as:

$$TG_R(x) = \min_{\xi \in U(x)} TG(\xi). \qquad (9)$$

As a result, the robust formulation of the topological gradient TG_R can be computed and used as a decision criterion on where the topology changes will take place.

III. NUMERICAL EXAMPLES

Fig. 1 shows a 3-phase, 4-pole single-barrier IPM motor. The model is solved using 2-D FEM in MagNet [8]. The periodic torque is obtained by solving the motor model for 30 rotor positions at one degree increments.

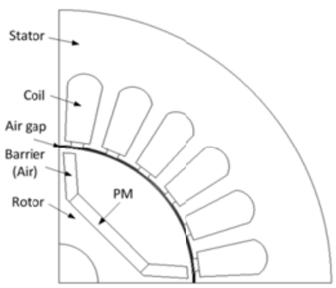

Fig. 1. Model of an IPM motor

A. *Design Example 1*

In this example, the design of the motor starts with an empty design space free of assumptions. The rotor core is made of nothing else but iron. The goal of the design is to determining the position and size of the permanent magnets in order to maximize the torque. The design region is the entire iron core of the rotor. Since torque is a function of the system co-energy, the state variables, A, are equal to the adjoint variables λ, and we do not need to solve the adjoint problem. The topological gradient is computed over the design region, and is plotted in fig. 2a.

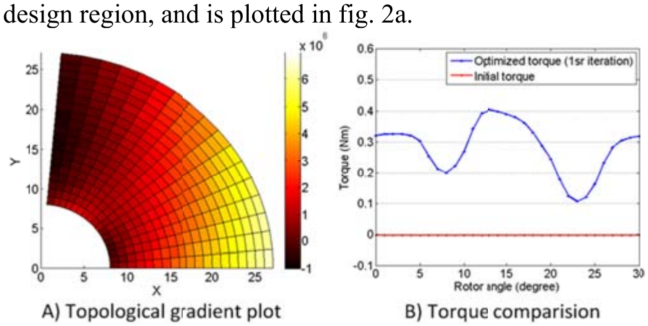

Fig. 2. Topology optimization to maximize torque

The region with the highest values of TG is shown in a light color. It corresponds to the type of motor which has a surface mounted PM. As a result, we have made a topology change by changing the material of this area from iron to PM. As a result, the output torque has increased significantly as shown in fig. 2b.

B. *Design Example 2*

The goal of design example 2 is to reduce the torque ripple. The objective function is defined as:

$$F = \sum_{i=0}^{30} \left(\frac{T_i - T_{avg}}{T_{avg}}\right)^2 . \qquad (10)$$

The TG is computed and shown in fig. 3a. The material in the highlighted area is changed from iron to air and the output torque waveform is plotted in fig. 3b. As we can see, the torque ripple has been reduced.

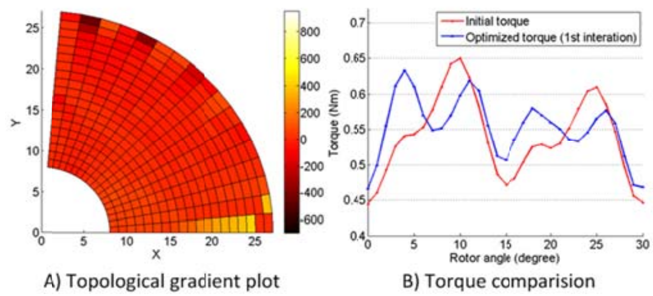

Fig. 3. Topology optimization to minimize torque ripple

IV. CONCLUSION

A topological gradient has been applied to the design of an IPM motor. Target function values were minimized through topological changes (adding a source or material swapping) suggested by the values of the TG. The robust design of the IPM will be discussed in the full paper.

V. REFERENCES

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